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Total coloring of Jahangir graph, Kragujevac tree and Dendrimers

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Abstract

Total coloring of a graph G is an assignment of colors to the elements (vertices and edges) of the graph with minimum number of colors such that no two adjacent or incident elements receive the same color. In this research paper, a study has been undertaken to obtain the total chromatic number of Jahangir graph $J_{n,m}$, Kragujevac tree $Kg_{d,k}$ and Dendrimers $POD_2[n]$, $T_{k,d}$ and $D_3(n)$.

Keywords: Jahangir graph, Kragujevac tree, Dendrimers, total coloring, total chromatic number.

Mathematics Subject Classification 2010: 05C05, 05C07, 05C15.

Introduction

A total coloring of a graph [16] is an assignment of colors to the elements (vertices and edges) of the graph such that no two adjacent or incident elements receive the same color. The minimum number of colors required for total coloring of G is referred to as total chromatic number and is denoted by $\chi_{tc}(G)$.

Bezahad [3] and Vizing [23] independently gave the total coloring conjecture between 1964 and 1968 in numerous occasions. They conjectured that the total chromatic number $\chi_{tc}(G)$ for any graph G satisfies the condition $\Delta(G)+1 \leq \chi_{tc}(G) \leq \Delta(G)+2$, where $\Delta(G)$ is the maximum degree of G and this is known as total coloring conjecture (TCC). The graph G is called type-1, if it is total colourable with $\Delta(G)+1$ colors and type-2 if total colourable with $\Delta(G)+2$ colors. This conjecture was verified further by Rosenfeld [19] and Vijayaaditya [22] for $\Delta(G)=3$ and by Kostochka [14] for $\Delta(G) \leq 5$.

TCC has been proved for various graphs. TCC for complete graph and complete multipartite graphs were verified by Behzad *et al* [4]. In [20], the total coloring of central graph of a path, a cycle and a star is discussed. Total coloring of closed Helm, Flower and

Bister graph Family are proved in [2]. The edge and total coloring of interval graphs were studied in [5]. Any graph G has a total coloring with at most Δ(G)+2 colors if Δ(G) ≥ 3/4 |V(G)|, which was studied in [10].

Thorny graph[24] : For any graph G* that can be obtained from a parent connected graph G by attaching $p_i \geq 0$ new vertices of degree one to each vertex i, is called thorny graph[7].

Motivated by the findings of all above mentioned authors, the study on "Total coloring of Jahangir graph, Kragujevac tree and Dendrimers" has been under taken. In this research paper we obtain the total chromatic number of Jahangir graph, Kragujevac tree and Dendrimers. The graphs considered in this study are simple, connected, undirected and finite. The undefined terminologies are followed from [9].

Remark [24] : If the graph is tree, total chromatic number of thorny graph of tree is $\Delta(T^{*}) + 1$.

Results

In this paper analytical methods are applied to obtain results on total coloring of Jahangir graph, Kragujevac tree and various types of Dendrimers.

Definition 1: Jahangir graph [1,17]

The Jahangir graph $J_{n,m}$ is a graph with (nm+1) vertices and (m(n+1)) edges, for all n ≥ 2 and for all m ≥ 3, i.e., a graph consisting of a cycle C_{nm} with one addition vertex which is adjacent to m vertices C_{nm} at distance n to each other on C_{nm}..

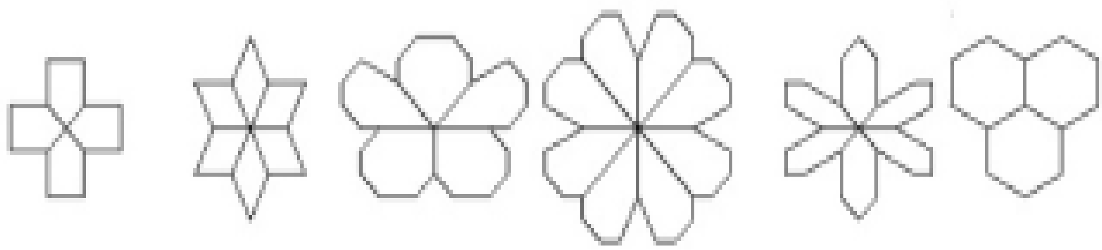

Figure 1. Jahangir graphs $J_{3,4}$, $J_{2,6}$, $J_{5,5}$, $J_{4,8}$, $J_{4,6}$ and $J_{4,3}$

Petals [13]: The region between the edges formed by joining central vertex to vertices on C_{nm} at the distance n from each other is known as a petal.

The degree of central vertex of $J_{n,m}$ is equal to the number of petals of $J_{n,m}$, where n is maximum distance of each petal from central vertex and m is the number of petals of $J_{n,m}$.

Theorem 1: Total chromatic number of Jahangir graph $J_{n,m,}$ for all $n \geq 2$ and for all $m \geq 3$ is given by

$$\chi_{tc}(J_{n,m}) = m+1$$

Proof: Let $J_{n,m}$ be the Jahangir graph, for all $n \geq 2$ and for all $m \geq 3$ with (nm+1) vertices and (m(n+1)) edges.

The maximum degree of $J_{n,m}$ is the degree of the central vertex i.e., $\Delta(J_{n,m}) = m$.

To begin with, let us color the central vertex of $J_{n,m}$ with one color . Then the edges which are incident to central vertex can be colored with m different colors.

i.e., Total number of colors required for coloring central vertex and edges incident to it are m+1.

The remaining edges of all petals form the path. As total coloring of odd or even length path requires only three colors and $m \geq 3$, m+1 colors are sufficient to color the remaining edges of petals irrespective of number of edges in the petals.

Therefore, the total chromatic number of $J_{n,m}$ =Number of petals +1.

$$\text{i.e.,} \chi_{tc}(J_{n,m,}) = m+1.$$

Definition 2: Kragujevac tree [8]

Let P_3 be the 3 vertex tree rooted at one of its terminal vertices. For k= 2, 3, …. . construct the rooted tree B_k by identifying the roots of k copies of P_3. The vertex obtained by identifying the roots of P_3 trees is the root of B_k.

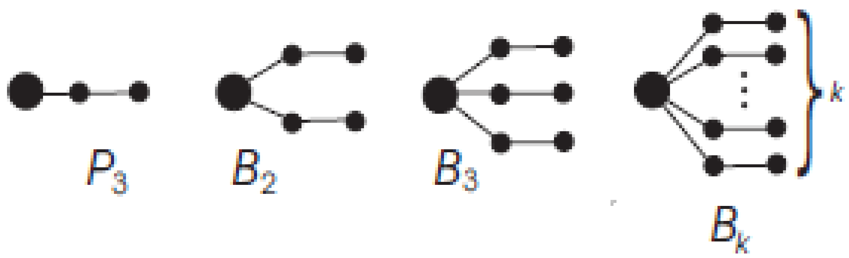

Figure 2 The structure of the rooted tree B_k

The rooted trees $B_{2,}$ B_3 and B_k obtained respectively by identifying the roots of 2, 3 and k copies of P_3. Their roots are indicated by large dots.

Let $d \geq 2$ be an integer. Let $\beta_{1,}$ $\beta_{2,}$……. $\beta_{d,}$ be rooted trees specified in definition 2, i.e., $\beta_{1,}$ $\beta_{2,}$…. $\beta_{d,} \in \{B_{2,} B_3, \ldots\}$. A Kragujevac tree T is a tree possessing a vertex of degree d, adjacent to the roots of $\beta_1, \beta_2, \ldots \beta_d$. This vertex is said to be the central vertex of T , where as d is the degree of T. The subgraphs $\beta_1, \beta_2, \ldots, \beta_d$ are the branches of T. The branches of T may be mutually isomorphic or non-isomorphic.

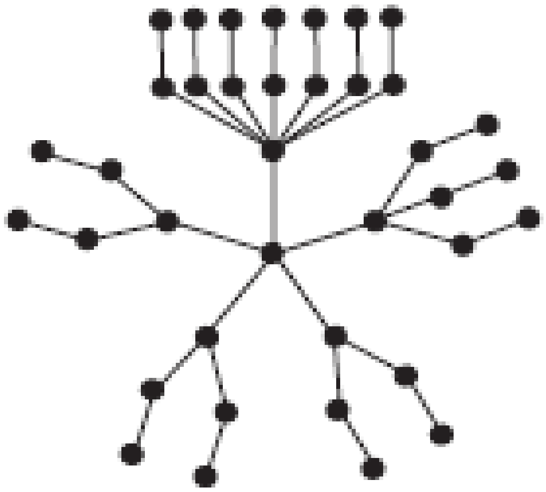

Figure 3. Kragujevac tree $Kg_{d,k}$

A typical Kragujevac tree is denoted by $Kg_{d,k}$, where $d \geq 2$ is the degree of central vertex and $k \geq 2$.

Theorem 2 : If $Kg_{d,k}$ be a Kragujevac tree of degree $d \geq 2$, then total chromatic number of Kragujevac tree, where $d > k$ and k is the highest branch rooted tree is given by

$$\chi_{tc}(Kg_{d,k}) = \Delta(Kg_{d,k}) + 1 = d + 1.$$

Proof : Let $Kg_{d,k}$ be a Kragujevac tree of degree $d \geq 2$ and $d > k$, where d is the degree of $Kg_{d,k}$ and k is the highest branch rooted tree.

Suppose d is the maximum degree of the central vertex of $Kg_{d,k}$, then d +1 colors are required to color the central vertex and edges incident to it, so that no vertices and edges receive the same color.

Next d+1 colors are sufficient for coloring the vertices and edges of branches of $Kg_{d,k}$, when $d > k$, where k is the highest branch rooted tree .

Hence the minimum colors required for total coloring of $Kg_{d,k}$ are d + 1.

$$\chi_{tc}(Kg_{d,k}) = \Delta(Kg_{d,k}) + 1 = d + 1.$$

Theorem 3: If $Kg_{d,k}$ be a Kragujevac tree of degree $d \geq 2$, then total chromatic number of Kragujevac tree, where $d \leq k$ and k is the highest branch rooted tree is given by

$$\chi_{tc}(Kg_{d,k}) = \Delta(B_k) + 2 = k + 2.$$

Proof: Let $Kg_{d,k}$ be a Kragujevac tree of degree $d \geq 2$ and $d \leq k$, where d is the degree of of $Kg_{d,k}$ and k is the highest branch rooted tree.

Suppose d is the maximum degree of the central vertex of $Kg_{d,k}$, then d +1 colors are required to color the central vertex and edges incident to it, so that no vertices and edges receive the same color.

But to color all the vertices and edges of branches B_k of $Kg_{d,k}$, when $d \leq k$, d+1 colors are not sufficient. As the number of branches increases, edges of branches of B_k are adjacent to each other. Hence k + 2 colors are required to color the edges and vertices of branches B_k.

Since $d \leq k$, k +2 colors (colors required for B_k) are required to color all the vertices and edges of $Kg_{d,k}$.

Hence minimum colors required for total coloring of Kragujevac tree $Kg_{d,k}$ are k +2.

$$\text{i.e.,} \quad \chi_{tc}(Kg_{d,k}) = \Delta(B_k) + 2 = k + 2.$$

Definition 3: Dendrimers [6]

Dendrimers are a new class of polymeric materials. The name of dendrimers is adapted from Greek word "Dendron", which refers to branches or trees. They are highly branched, mono-disperse macromolecules. Dendrimers are classified by "generation" i.e., how many branching layers they comprise of. The size and numbers of atoms in a dendrimer are significantly increasing with the growing generation.

In this section we obtain the total coloring of POPAM, $T_{k,d}$ and Nanostar dendrimers.

Poly-Propylene Amine (POPAM) dendrimer [11]:

POPAM dendrimers is denoted by $POD_2[n]$, where $n \geq 0$, then $POD_2[n]$ dendrimer contains 2^{n+5}-10 vertices and 2^{n+5}-11 edges.

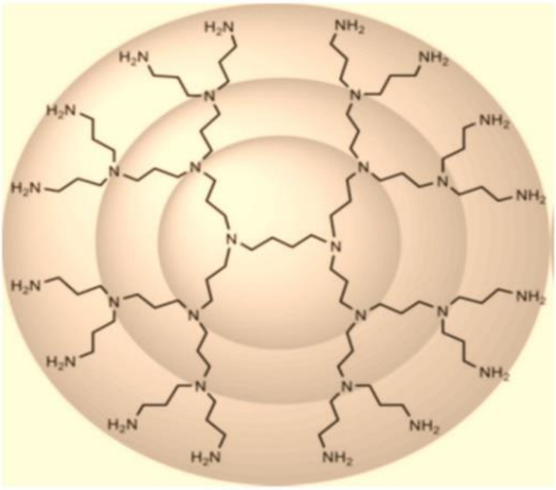

Figure 4. POPAM dendrimer of generations G_n with two growth stages, POD2[n]

Theorem 4: Total chromatic number of POPAM dendrimer POD2[n], $n \geq 0$, is given by

$$\chi_{tc}(POD2[n]) = \Delta(POD2[n])+1=3+1=4$$

Proof: Let POD2[n] be the POPAM dendrimer with n growth, $n \geq 0$, with 2^{n+5}-10 vertices and 2^{n+5}-11 edges.

In particular, POD2[n] is dendrimer with 2-growth with118 vertices and 117 edges.

The maximum degree of POD2[n] is the total number of edges incident at the vertex of the 0^{th} growth.

i.e., Max. degree of Δ(POD2[n]) =3

We can assign one color to the central vertex and three different clolors to the edges incident to this central vertex so that no two incident or adjacent elements receive the same color.

i.e., Centtral vertex and its incident edges can be colored with 4 different colors.

Further edges in the 1^{st} and 2^{nd} growth of POD2 [n] form the tree with maximum degree 3. From the remark A, the total chromatic number of the tree is $\Delta(T)$ +1.

Hence 4 colors are sufficient to color the remaining vertices and edges of 1^{st} and 2^{nd} growth of POD2 [n].

Therefore, minimum number of colors required for coloring the graph POD2 [n] are 4 so that no two adjacent or incident elements receive the same color.

$$\text{Hence,} \quad \chi_{tc}(POD2[n]) = \Delta(POD2[n])+1=4$$

$T_{k,d}$ Dendrimer[12, 21]

A regular dendrimer $T_{k,d}$, $k \geq 0$, $d \geq 2$ is a central tree with a centre vertex and having every non-pendent vertex of degree d and the distance from centre vertex to each pendent vertex is k.

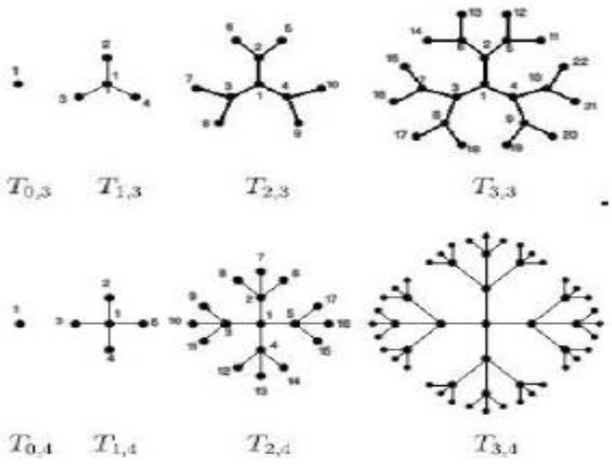

Figure 5. $T_{k,d}$ Dendrimer

Theorem 5: Total chromatic number of dendrimer $T_{k,d}$, $k \geq 0$, $d \geq 2$ is given by

$$\chi_{tc}(T_{k,d},) = \Delta(T_{k,d}]) + 1 = d + 1.$$

Proof: Let $T_{k,d}$ be a regular dendrimer with $k \geq 0$, $d \geq 2$, where d is the degree of the central vertex of $T_{k,d}$ and k is the distance from centeral vertex to each pendent vertex.

Let us start the coloring of $T_{k,d}$ with the central vertex. Suppose d is the maximum degree central vertex of $T_{k,d}$. Obviously d +1 different colors are required for coloring of central vertex and edges incident to it.

Further dendrimer $T_{k,d}$ grows with more number of branches and forms regular tree with maximum degree d. Then by remark A, total chromatic number of tree is $\Delta(T) + 1$.

Thus minimum number of colors required for toal coloring of $T_{k,d}$ are d +1.

$$\text{Hence } \chi_{tc}(T_{k,d},) = \Delta(T_{k,d}])+1=d+1.$$

Nanostar dendrimer $D_3(n)$[15, 18]

Let $D_3[n]$ is a n^{th} growth of nanostar dendrimer, for all $n \in N$. It has $|V(D_3(n)|=21(2^{n+1})-20$ number of vertices and $|E(D_3[n])| = 24(2^{n+1}-1)$ edges.

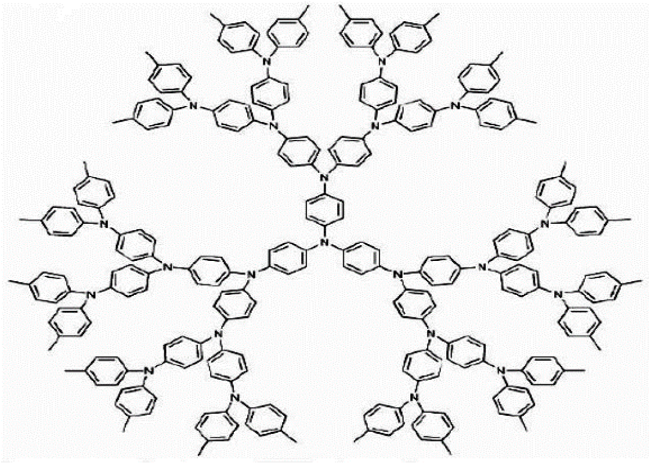

Figure 6 Nanostar dendrimer D_3(n)

Theorem 6: Total chromatic number of Nanostar dendrimer $D_3[n]$, $n \in N$, is given by

$$\chi_{tc}(D_3[n]) = \Delta(D_3[n]) + 1 = 3 + 1 = 4.$$

Proof: Let $D_3[n]$ is a n^{th} growth nanostar dendrimer.

The maximum degree of $D_3[n]$ i. e., $\Delta(D_3[n])$ is 3 . At 0^{th} level the colors required for central vertex and edges incident to central vertex are 4 (1 for central vertex and 3 for edges). As the dendrimer grows to successive levels i.e., 1^{st} , 2^{nd} ……n^{th} levels, there is an increase in the number of vertices and edges with maximum degree 3. Irrespective of the levels . vertices and edges of $D_3[n]$, $\Delta(D_3[n]) + 1 = 4$ colors are sufficient to color $D_3[n]$.

Hence toal chromatic number of Nanostar dendrimer $D_3[n]$ is

$$\chi_{tc}(D_3[n]) = \Delta(D_3[n]) + 1 = 3 + 1 = 4.$$

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